

Homework Week 6

MATHEMATICS OF DEEP LEARNING
MASH & IASD 2025

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Instructions: This homework is **due on Monday 03/03/2025**. Please send your solutions in a Python Jupyter notebook file named HW6_NOM_PRENOM.IPYNB to the above address with the subject “[MATHSDL2025] Homework 6”.

Packages allowed: Numpy, Matplotlib and sklearn. Notebooks with running errors will not be considered.

1 Exercises

Exercise 1.

Reproduce Figure 1 (left & right) from the lecture notes. Your notebook must show both theoretical curves (solid lines) obtained by solving the self-consistent equations and the simulations (crosses) obtaining by evaluating the ridge estimator, both discussed during the lectures.

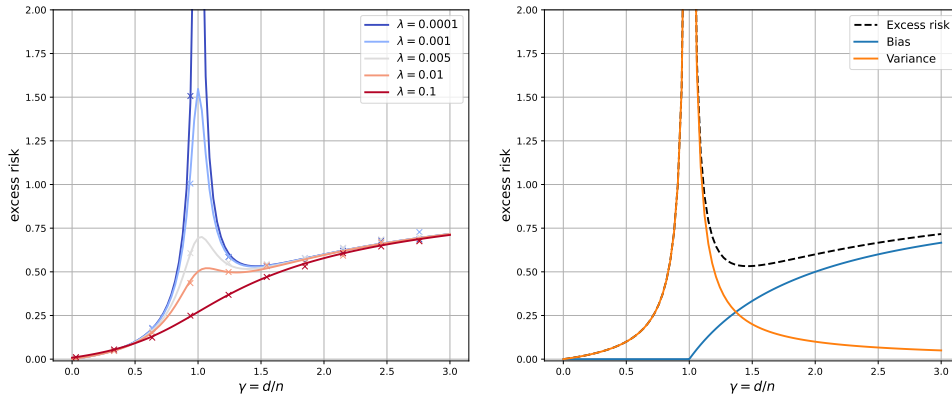


Figure 1: Excess risk of ridge regression as a function of $\gamma = d/n$ for $\sigma^2 = 0.1$, $\|\theta_\star\|_2^2 = 1$ and isotropic covariates $\Sigma = \mathbf{I}_d$. **(Left)** Increasing values of λ . **(Right)** Bias-variance decomposition of the excess risk for the ridge interpolator $\lambda = 0^+$ (a.k.a. ordinary least-squares estimator).

Exercise 2.

Reproduce Figure 2 from the lecture notes. Your notebook must show both theoretical curves (solid lines) obtained by solving the self-consistent equations and the simulations (crosses) obtaining by evaluating the ridge estimator, both discussed during the lectures.

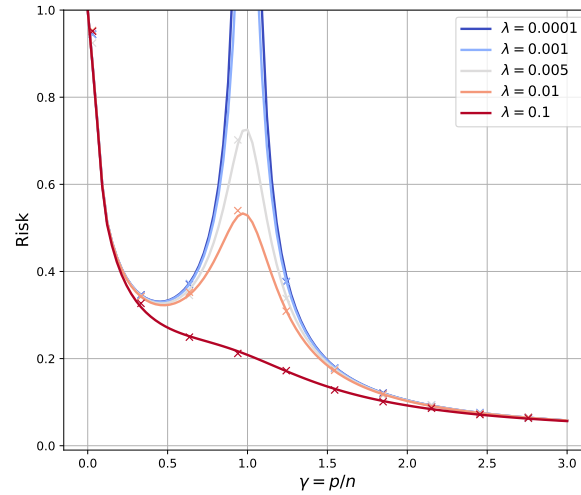


Figure 2: Risk of ridge regression as a function of $\gamma = p/n$ for the latent space model defined in Section 3.4 of the notes for $\tau^2 = 0$, $\alpha = n/d = 10$. Solid curves show the theoretical result, obtained from solving the self-consistent eq.(3.54), and crosses are finite size simulations with $d = 100$.