Homework Week 5

MATHEMATICS OF DEEP LEARNING
MASH & IASD 2025

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Instructions: This homework is **due on Monday 24/02/2025**. Please send your solutions in a Python Jupyter notebook file named HW5_NOM_PRENOM.IPYNB to the above address with the subject "*[MATHSDL2025] Homework 5*".

Packages allowed: Numpy and Matplotlib. Notebooks with running errors will not be considered.

1 Exercises

Exercise 1.

Reproduce Figure 1 from the lecture notes. Your notebook must show both the histogram of the empirical spectral density and the analytical curve of the semi-circle law discussed during the lectures.

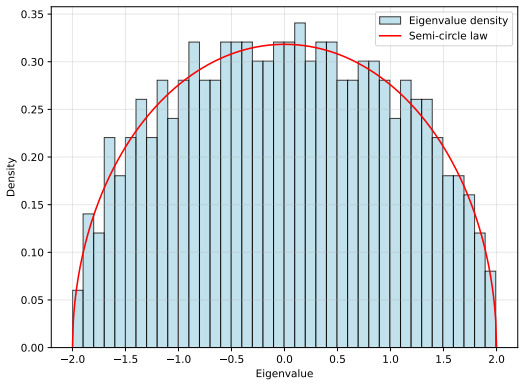

Figure 1: Histogram of eigenvalues of a GOE matrix of dimension $d = 500$ with 40 bins. The red solid curve denotes de Wigner semi-circle law $\mu(\mathrm{d}x) = \frac{\sqrt{4-x^2}}{2\pi}\mathbf{1}_{[-2,2]}\mathrm{d}x$.

Exercise 2.

Reproduce Figures 2 (left) and 2 (right) from the lecture notes. Your notebook must show both the histogram of the empirical spectral density and the analytical curve of the Marchenko Pastur law discussed during the lectures.

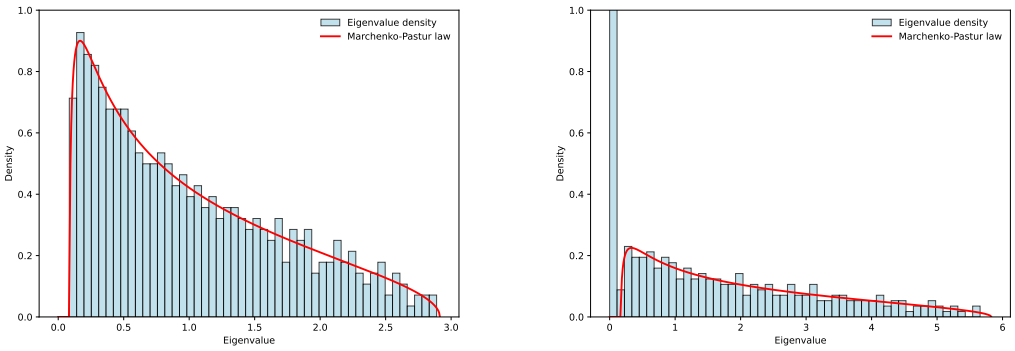

Figure 2: Histogram of eigenvalues of a Wishart matrix of dimension $d = 500$ with 40 bins for $\gamma = 0.5$ (**left**) and $\gamma = 2$ (**right**). The red solid curve denotes de Marchenko-Pastur law given by $\mu_{\mathrm{mp}}(\mathrm{d}x) = \left(1 - \frac{1}{\gamma}\right)_+ \delta_0 + \frac{\sqrt{(\gamma_+ - x)(x - \gamma_-)}}{2\pi\gamma x} \mathbf{1}_{[\gamma_-, \gamma_+]}(x)\mathrm{d}x0$.