

Homework Week 3

MATHEMATICS OF DEEP LEARNING
MASH & IASD 2025

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Instructions: This homework is **due on Monday 03/02/2025**. Please send your solutions in a PDF file named HW3_NOM_PRENOM.PDF to the above address with the subject “[MATHSDL2025] Homework 3”. Formats accepted: LaTeX or a **readable** scan of handwritten solutions.

1 Exercises

Exercise 1.

By following the same steps as in the proof of Lemma 1 from the notes, prove that $\mathcal{F}_{\text{exp},d}$ are universal approximators over $[0, 1]^d$

Exercise 2.

- (a) Consider a Gaussian density with variance σ^2 :

$$f(\mathbf{x}) = \frac{1}{(2\pi\sigma^2)^{d/2}} e^{-\frac{1}{2\sigma^2}\|\mathbf{x}\|_2^2} \quad (1)$$

Show that its Barron norm is given by:

$$\|f\|_B = \frac{1}{\sqrt{\pi}} \frac{\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d}{2})} \frac{1}{(2\pi\sigma^2)^{\frac{d+1}{2}}} \quad (2)$$

Conclude that for $\sigma^2 \geq 1$, we have $\|f\|_B = O(\sqrt{d})$ as $d \rightarrow \infty$. What happens for $\sigma^2 < 1$?

- (b) Show that for the ridge function $f(\mathbf{x}) = \sigma(\langle \mathbf{w}, \mathbf{x} + b \rangle)$ the Barron norm is bounded by:

$$\|f\|_B \leq \|\mathbf{w}\|_2 \int_{\mathbb{R}} |\xi \hat{\sigma}(\xi)| d\xi. \quad (3)$$

Conclude that the ridge function with sigmoid-like activation are in \mathcal{F}_B , but not with ReLU activation $\sigma(x) = x_+$.