Homework Week 3

MATHEMATICS OF DEEP LEARNING
MASH & IASD 2026

Lecturer: Bruno Loureiro, bruno.loureiro@di.ens.fr

Instructions: This homework is **due on Monday 09/02/2026**. Please upload your solutions in a PDF file named HW3_NOM_PRENOM.PDF here. Formats accepted: PDF (LaTeX or a **readable** scan of handwritten solutions).

1 Exercises

Exercise 1.

By following the same steps as in the proof of Lemma 1 from the notes, prove that $\mathcal{F}_{\exp,d}$ are universal approximators over $[0,1]^d$

Exercise 2.

(a) Consider a Gaussian density with variance σ^2:

$$f(\boldsymbol{x}) = \frac{1}{(2\pi\sigma^2)^{d/2}} e^{-\frac{1}{2\sigma^2}||\boldsymbol{x}||_2^2} \tag{1}$$

Show that its Barron norm is given by:

$$||f||_B = \frac{1}{\sqrt{\pi}} \frac{\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d}{2})} \frac{1}{(2\pi\sigma^2)^{\frac{d+1}{2}}} \tag{2}$$

Conclude that for $\sigma^2 \geq 1$, we have $||f||_B = O(\sqrt{d})$ as $d \to \infty$. What happens for $\sigma^2 < 1$?

(b) Show that for the ridge function $f(\boldsymbol{x}) = \sigma(\langle \boldsymbol{w}, \boldsymbol{x} + b)$ the Barron norm is bounded by:

$$||f||_B \leq ||\boldsymbol{w}||_2 \int_{\mathbb{R}} |\xi \hat{\sigma}(\xi)| \mathrm{d}\xi. \tag{3}$$

Conclude that the ridge function with sigmoid-like activation are in $\mathcal{F}_B$, but not with ReLU activation $\sigma(x) = x_+$.